III. Plasma Lens

The physics of the intense particle beams has a great impact not only on the basic research but also the applications in medicine and industry. The plasma lens is a device to provide an ultra-strong final focus at the end of the linear colliders. To appreciate the possibilities of the plasma lens it is quite natural to compare this with the usual magnetic and electrostatic lenses. In magnetic lenses, focusing capability is proportional to the magnetic field gradient. The practical upper limit of the quadrupole focusing lens is in the order of 10^2 T/m, while for plasma lens having density of 10^{17} cm^{-3}, its focusing capability is equivalent to a magnetic field gradient of 3×10^6 T/m (about four orders of magnitude more than that of a magnetic quadrupole lens).

In what follows, we will illuminate why the intense relativistic particle beams could produce self-focusing beams and do not blow themselves apart in free space.

a) Consider a long cylindrical electron beam of uniform number density n and average speed v (both quantities in laboratory frame). Derive the expression for the electric field at a point at distance r from the central axis of the beam using classical electromagnetics. (1 point)

b) Derive the expression for the magnetic field at the same point as in a). (2 points)

c) What is then the net outward force on the electron in the electron beam passing that point? (1 point)

d) Assuming that the expression obtained in c) is applicable at relativistic velocities, what will be the force on the electron as v approaches speed of light c, where $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$? (1 point)

e) If the electron beam of radius R enters into a plasma of uniform density $n_0 < n$ (the plasma is an ionized gas of ions and electrons with equal charge density), what will be the net force on the *stationary plasma ion* at distance r' outside the electron beam long after the beam entering the plasma. You may assume that the density of the plasma ions remains constant and the cylindrical symmetry is maintained. (3 points)

f) After long enough time, what is the net force on an electron at distance r from the central axis of the beam in the plasma, assuming $v \to c$ provided that the density of the plasma ions remains constant and the cylindrical symmetry is maintained? (2 points)